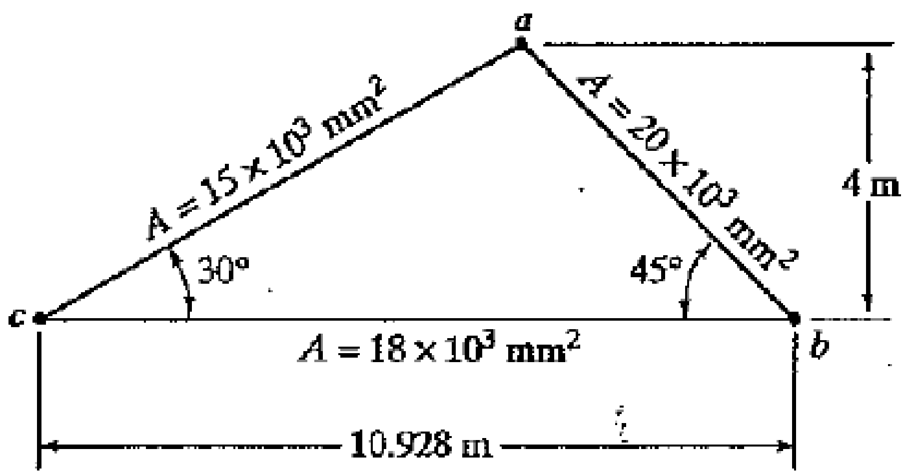
Ejercicios resueltos

Problema 3.1

For the system shown:

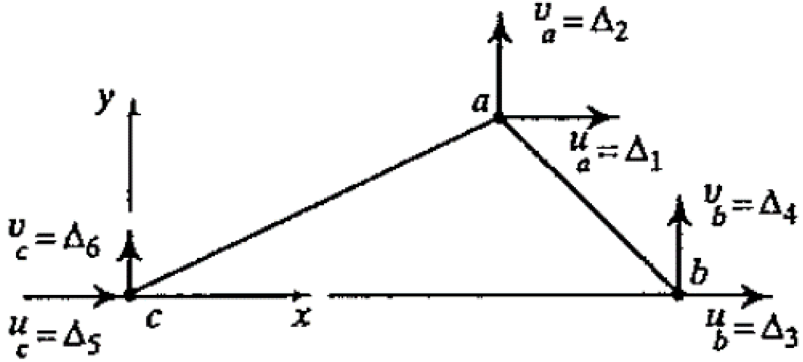
1. Write the member force-displacement relationships in global coordinates.
2. Assemble the global stiffness equations.
3. Show that the global stiffness equations contain rigid-body-motion terms. $E = 200{,}000$ MPa.

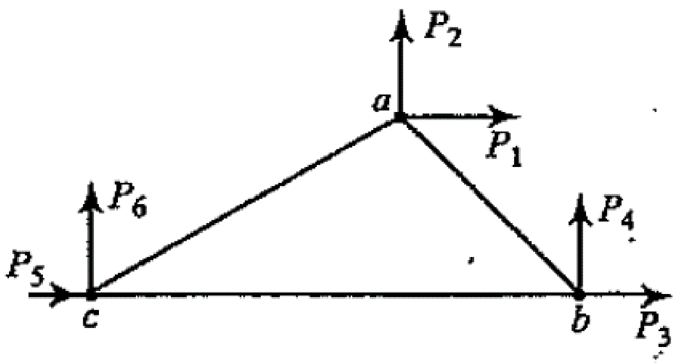


Ejemplos

Solución

Defina las coordenadas, grados de libertas y fuerzas externas como sigue

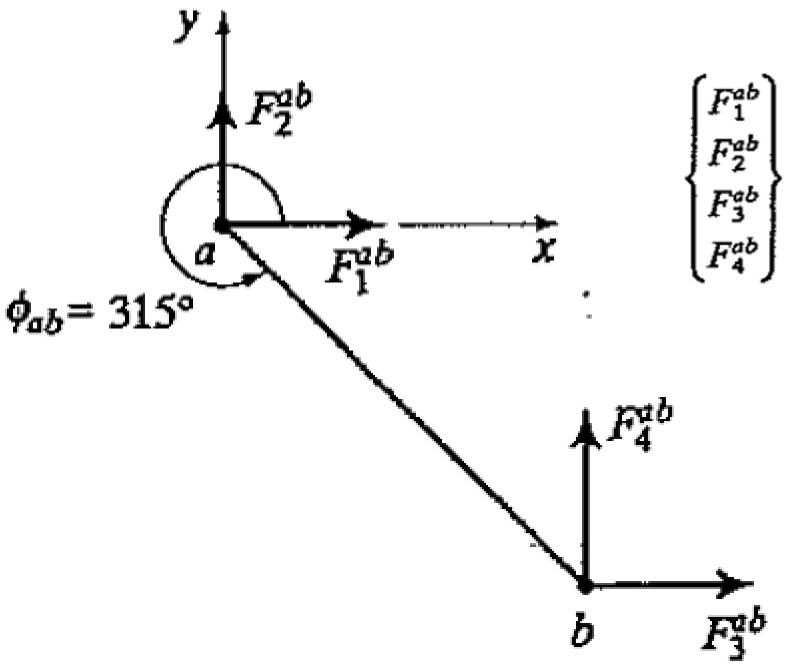




Ejemplos

1. Relaciones fuerza-desplazamiento en las barras

Barra ***ab***



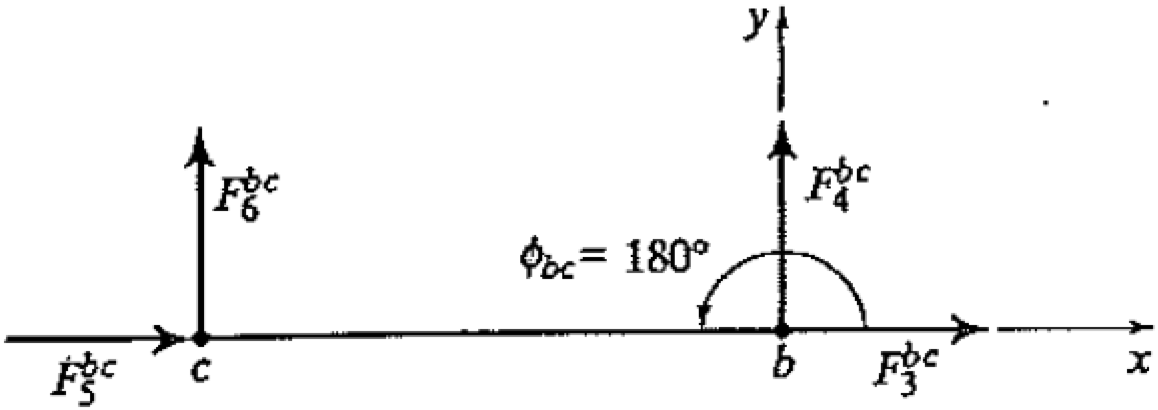
$$\left(\frac{EA}{L}\right)_{ab} = \frac{200 \times 20 \times 10^3}{4\sqrt{2} \times 10^3} = 707.11 \text{ kN/mm}$$

$$\begin{Bmatrix} F_1^{ab} \\ F_2^{ab} \\ F_3^{ab} \\ F_4^{ab} \end{Bmatrix} = 707.11 \begin{bmatrix} 0.500 & -0.500 & -0.500 & 0.500 \\ & 0.500 & 0.500 & -0.500 \\ & & 0.500 & -0.500 \\ \text{Sym.} & & & 0.500 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix}$$

Ejemplos

Relaciones fuerza-desplazamiento en las barras

Barra ***bc***



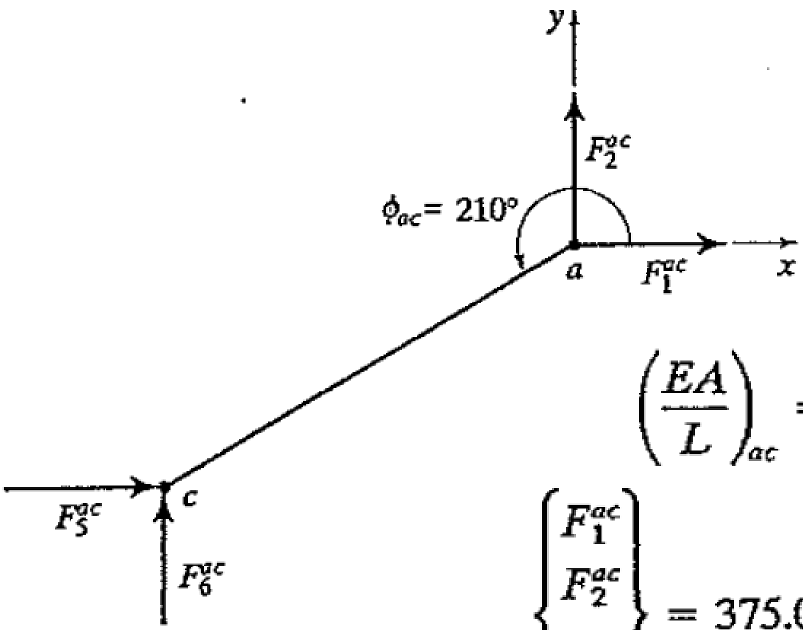
$$\left(\frac{EA}{L}\right)_{bc} = \frac{200 \times 18 \times 10^3}{10.928 \times 10^3} = 329.43 \text{ kN/mm}$$

$$\begin{Bmatrix} F_3^{bc} \\ F_4^{bc} \\ F_5^{bc} \\ F_6^{bc} \end{Bmatrix} = 329.43 \begin{bmatrix} 1.000 & 0 & -1.000 & 0 \\ & 0 & 0 & 0 \\ & & 1.000 & 0 \\ \text{Sym.} & & & 0 \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix}$$

Ejemplos

Relaciones fuerza-desplazamiento en las barras

Barra ***ac***



$$\left(\frac{EA}{L}\right)_{ac} = \frac{200 \times 15 \times 10^3}{8 \times 10^3} = 375.00 \text{ kN/mm}$$

$$\begin{Bmatrix} F_1^{ac} \\ F_2^{ac} \\ F_5^{ac} \\ F_6^{ac} \end{Bmatrix} = 375.00 \begin{bmatrix} 0.750 & 0.433 & -0.750 & -0.433 \\ & 0.250 & -0.433 & -0.250 \\ & & 0.750 & 0.433 \\ \text{Sym.} & & & 0.250 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix}$$

Ejemplos

2. Matriz de rigidez global

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{Bmatrix} = 10^2 \begin{bmatrix} 6.348 & -1.912 & -3.536 & 3.536 & -2.812 & -1.624 \\ & 4.473 & 3.536 & -3.536 & -1.624 & -0.938 \\ & & 6.830 & -3.536 & -3.294 & 0 \\ & \text{Sym.} & & 3.536 & 0 & 0 \\ & & & & 6.107 & 1.624 \\ & & & & & 0.938 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix}$$

3. Movimiento del cuerpo rígido. Añadiendo filas 1 y 3 de la matriz de rigidez global forma el vector

$$\lfloor 2.812 \quad 1.624 \quad 3.294 \quad 0 \quad -6.107 \quad -1.624 \rfloor$$

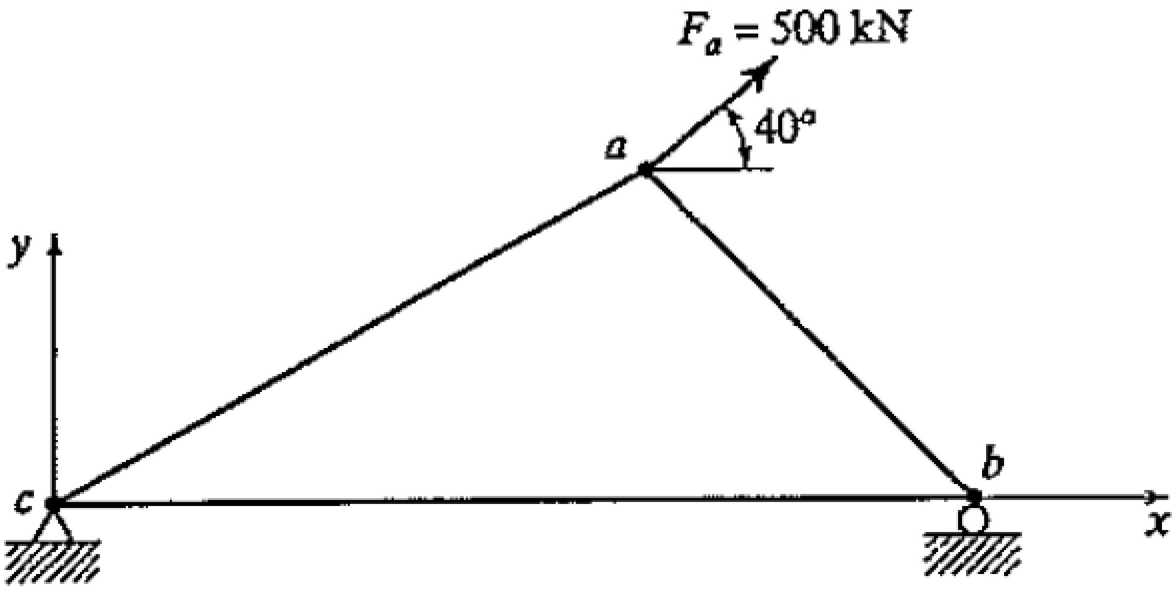
which is the negative of row 5. Therefore, there is linear dependence, the determinant is zero, and the matrix is singular. This is a signal that, under an arbitrary load, the displacements are indefinite; that is, there may be rigid body motion.

Ejemplos

Problema 3.2

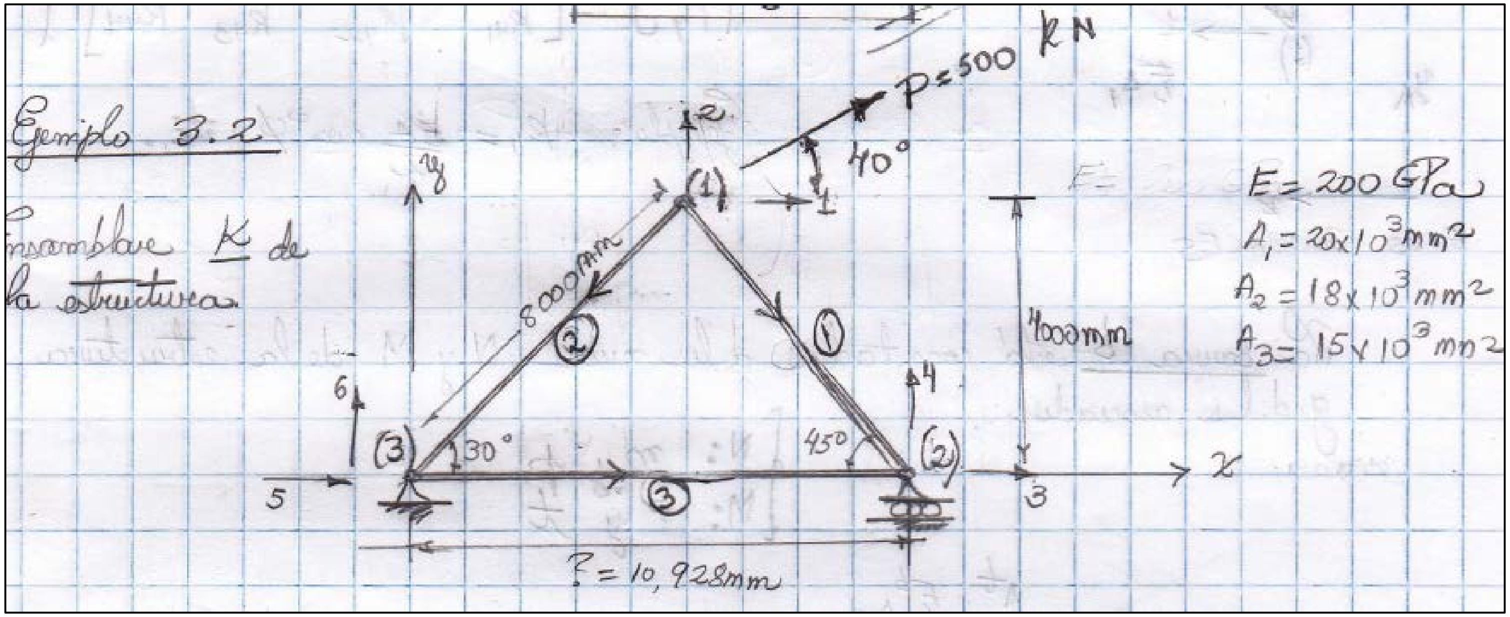
The truss of Example 3.1 is supported and loaded as shown.

1. Calculate the displacements at *a* and *b*.
2. Calculate the reactions.
3. Calculate the bar forces. Use equations of Example 3.1.

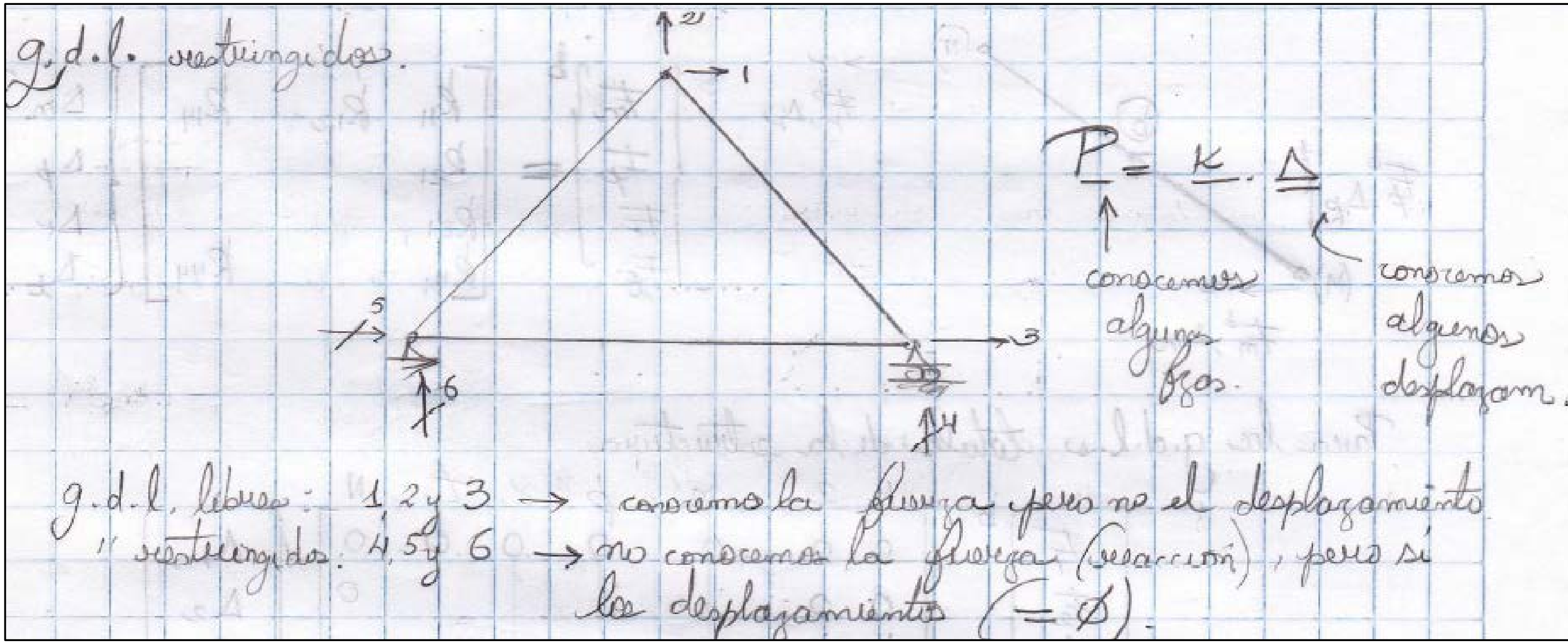


Ejemplos

Solución



Ejemplos


g.d.l. restringidos.
2
1
5
3
6
4
P = K . Δ
conocemos algunas fzas.
conocemos algunos desplazam.
g.d.l. libres: 1, 2 y 3 → conocemos la fuerza pero no el desplazamiento
" restringidos: 4, 5 y 6 → no conocemos la fuerza (reacción), pero sí los desplazamientos (= ∅).

Ejemplos

Particionamos los g.d.l. en libres y restringidos. La ecuac. de equilibrio global resulta:

$$\underline{P} = \left\{ \begin{array}{c} P_f \\ \hdashline P_s \end{array} \right\}$$

f: libre (free)

s: restringido (supported)

$$\Rightarrow \quad \underline{P} = \left\{ \begin{array}{c} P_f \\ \hdashline P_s \end{array} \right\} = \left[\begin{array}{c:c} K_{ff} & K_{fs} \\ \hdashline K_{sf} & K_{ss} \end{array} \right] \left\{ \begin{array}{c} \Delta_f \\ \hdashline \Delta_s \end{array} \right\}$$

$$\hphantom{\Rightarrow \quad \underline{P} = \{\}=[} \underbrace{\hphantom{K_{sf}}}_{f} \quad \underbrace{\hphantom{K_{ss}}}_{s}$$

Para el ejemplo:

$$\left\{ \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \hline P_4 \\ P_5 \\ P_6 \end{array} \right\} = \left\{ \begin{array}{c} P_f \\ \hdashline P_s \end{array} \right\} = \left\{ \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ \hline R_4 \\ R_5 \\ R_6 \end{array} \right\}$$

Ejemplos

Además:

$$\underline{\Delta} = \left\{\begin{array}{c} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \hline \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{array}\right\} = \left\{\begin{array}{c} \Delta_f \\ \hline \Delta_s \end{array}\right\} = \left\{\begin{array}{c} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \hline 0 \\ 0 \\ 0 \end{array}\right\}$$

Entonces:

$$\left\{\begin{array}{c} P_f \\ P_s \end{array}\right\} = \begin{bmatrix} K_{ff} & K_{fs} \\ K_{sf} & K_{ss} \end{bmatrix} \left\{\begin{array}{c} \Delta_f \\ 0 \end{array}\right\}$$

$$\Rightarrow \underline{P}_f = \underline{K}_{ff} \cdot \underline{\Delta}_f + \underline{K}_{fs} \cdot \underline{0} \quad \ldots (1) \quad \Rightarrow \underline{P}_f = \underline{K}_{ff} \cdot \underline{\Delta}_f$$

$$\underline{P}_s = \underline{K}_{sf} \cdot \underline{\Delta}_f + \underline{K}_{ss} \cdot \underline{0} \quad \ldots (2) \qquad \therefore \boxed{\underline{\Delta}_f = \underline{K}_{ff}^{-1} \cdot \underline{P}_f}$$

$\underline{\Delta}_f$: desplazam. en gdl libres

$\underline{\Delta} = [\underline{\Delta}_f \ \underline{\Delta}_s]^T$ está completamente definido.

Ejemplos

De (2) calculamos las reacciones:

$$\boxed{\underline{P}_{s} = \underline{K}_{sf} \cdot \underline{\Delta}_{f}}$$

Las fuerzas en las barras se calcula con:

$$\underline{F}^{e} = \underline{k}^{e} \cdot \underline{\Delta}^{e}$$

extraemos de $\underline{\Delta}$ de acuerdo a la conectividad.

$$\Rightarrow \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{Bmatrix}_b = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix}_b$$

Solución con MATLAB

```
% Solución el ejemplo 3.2 del libro
% a) Matriz de rigidez de cada barra

MatRig = zeros(4,4,3); % matriz 3D para almacenar matrices
GDL    = zeros(3, 4);  % matriz de grados de libertad
                       % para 3 barra y 4 g.d.l. x barra
E = 200;
% Barra 1
A1 = 20000; L1 = 4000 / sin(45*pi/180);
phi1 = -45;
k1   = RigArma2D(E, A1, L1, phi1);
MatRig(:,:,1) = k1;
GDL(1,:) = [1 2 3 4];

% Barra 2
A2 = 15000; L2 = 4000 / sin(30*pi/180);
phi2 = 210;
k2   = RigArma2D(E, A2, L2, phi2);
MatRig(:,:,2) = k2;
GDL(2,:) = [1 2 5 6];

% Barra 3
A3 = 18000; L3 = 10928;
phi3 = 180;
k3   = RigArma2D(E, A3, L3, phi3);
MatRig(:,:,3) = k3;
GDL(3,:) = [ 3 4 5 6 ];
```

Solución con MATLAB

```
% b) Matriz de rigidez global de la estructura
K = zeros(6,6);
for n = 1 : 3
    mat = MatRig(:,:,n);
    gdl = GDL(n,:);   % g.d.l que le corresponde a la barra n
    for i = 1 : 4
        for j = 1:4
            fil = gdl(i);
            col = gdl(j);
            K(fil,col) = K(fil,col) + mat(i,j);
        end
    end
end
 % Ejercicio 3.2 del libro
P  = 500; angr = 40*pi/180; % carga aplicada
P1 = P*cos(angr); P2 = P*sin(angr); P3 = 0; % en los g.d.l.
Pf = [ P1 P2 P3]';  % cargas en los g.d.l. libres
Kff = K(1:3, 1:3);
Ksf = K(4:6, 1:3);
Kfs = K(1:3, 4:6);  % Tambien Kfs = Ksf'
Kss = K(4:6, 4:6);
Df  = inv(Kff)*Pf % Desplazamientos en los g.d.l. libres
Ps  = Ksf*Df      % Reacciones en los g.d.l. restringidos
```

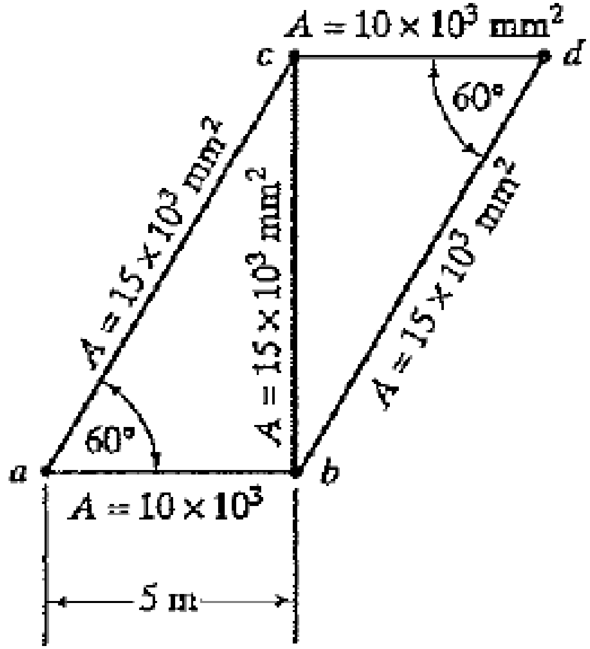
Ejemplos

Problema 3.4

For the system shown:

1. Write the force-displacement relationships in global coordinates.
2. Assemble the global stiffness equations.
3. Show that the stiffness equations contain rigid-body-motion terms.

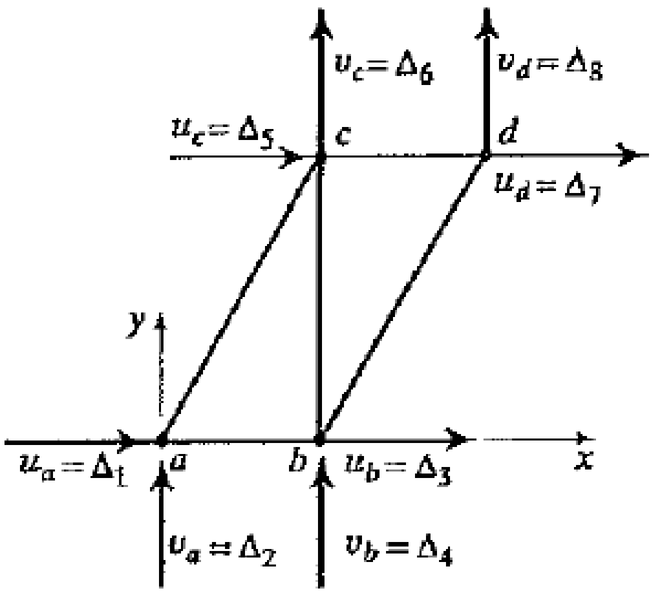
$E = 200{,}000$ MPa.

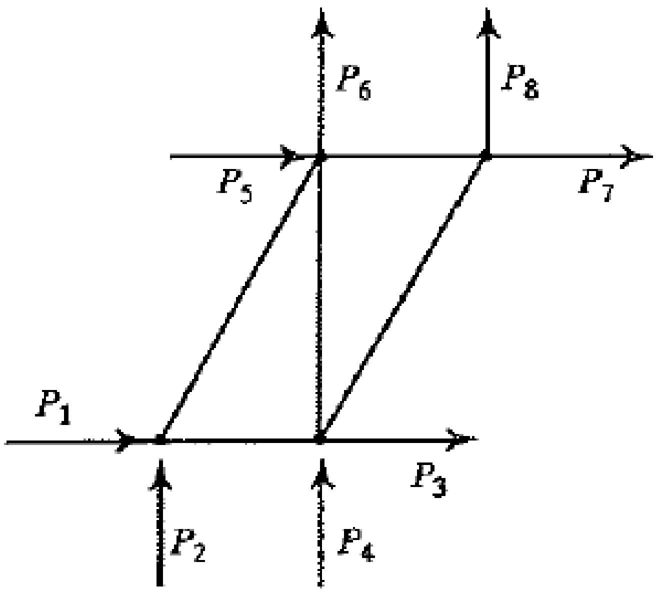


Ejemplos

Solución

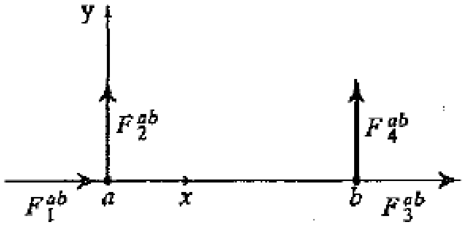
Defina las coordenadas, grados de libertas y fuerzas externas como sigue





Ejemplos

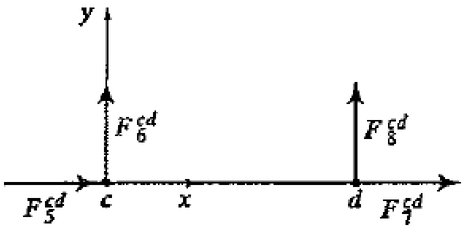
1. Member force-displacement relationships (see Equation 2.5):
   Member *ab*



$$\left(\frac{EA}{L}\right)_{ab} = \frac{200 \times 10 \times 10^3}{5 \times 10^3} = 400 \text{ kN/mm}$$

$$\begin{Bmatrix} F_1^{ab} \\ F_2^{ab} \\ F_3^{ab} \\ F_4^{ab} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix} = 400.00 \begin{bmatrix} 1.000 & 0 & -1.000 & 0 \\ & 0 & 0 & 0 \\ & & 1.000 & 0 \\ \text{Sym.} & & & 0 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \end{Bmatrix}$$

Member *cd*



$$\left(\frac{EA}{L}\right)_{cd} = 400 \text{ kN/mm}$$

$$\begin{Bmatrix} F_5^{cd} \\ F_6^{cd} \\ F_7^{cd} \\ F_8^{cd} \end{Bmatrix} = \begin{bmatrix} k_{55} & k_{56} & k_{57} & k_{58} \\ k_{65} & k_{66} & k_{67} & k_{68} \\ k_{75} & k_{76} & k_{77} & k_{78} \\ k_{85} & k_{86} & k_{87} & k_{88} \end{bmatrix} \begin{Bmatrix} \Delta_5 \\ \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix} = 400.00 \begin{bmatrix} 1.000 & 0 & -1.000 & 0 \\ & 0 & 0 & 0 \\ & & 1.000 & 0 \\ \text{Sym.} & & & 0 \end{bmatrix} \begin{Bmatrix} \Delta_5 \\ \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix}$$

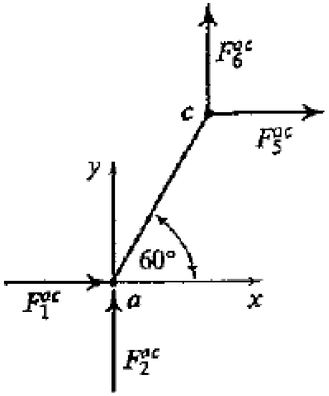
Ejemplos

Member *ac*

$$\left(\frac{EA}{L}\right)_{ac} = \frac{200 \times 15 \times 10^3}{10 \times 10^3} = 300 \text{ kN/mm}$$

$$\begin{Bmatrix} F_1^{ac} \\ F_2^{ac} \\ F_5^{ac} \\ F_6^{ac} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{15} & k_{16} \\ k_{21} & k_{22} & k_{25} & k_{26} \\ k_{51} & k_{52} & k_{55} & k_{56} \\ k_{61} & k_{62} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix}$$

$$= 300.00 \begin{bmatrix} 0.250 & 0.433 & -0.250 & -0.433 \\ & 0.750 & -4.333 & -0.750 \\ & & 0.250 & 0.433 \\ \text{Sym.} & & & 0.750 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix}$$



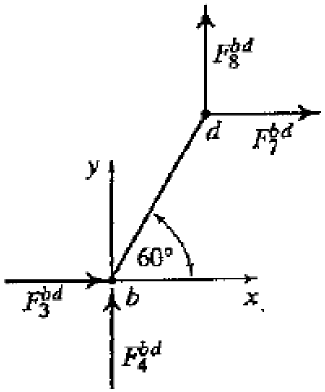
Ejemplos

Member *bd*

$$\left(\frac{EA}{L}\right)_{bd} = 300 \text{ kN/mm}$$

$$\begin{Bmatrix} F_3^{bd} \\ F_4^{bd} \\ F_7^{bd} \\ F_8^{bd} \end{Bmatrix} = \begin{bmatrix} k_{33} & k_{34} & k_{37} & k_{38} \\ k_{43} & k_{44} & k_{47} & k_{48} \\ k_{73} & k_{74} & k_{77} & k_{78} \\ k_{83} & k_{84} & k_{87} & k_{88} \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_4 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix}$$

$$= 300.00 \begin{bmatrix} 0.250 & 0.433 & -0.250 & -0.433 \\ & 0.750 & -0.433 & -0.750 \\ & & 0.250 & 0.433 \\ \text{Sym.} & & & 0.750 \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_4 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix}$$

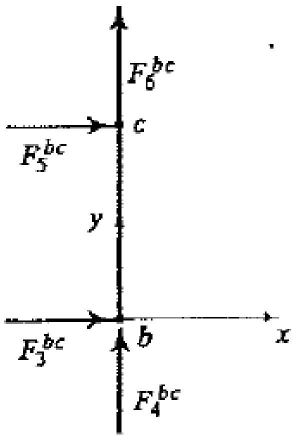


Ejemplos

Member bc

$$\left(\frac{EA}{L}\right)_{bc} = \frac{200 \times 15 \times 10^3}{5\sqrt{3} \times 10^3} = 346.41 \text{ kN/mm}$$

$$\begin{Bmatrix} F_3^{bc} \\ F_4^{bc} \\ F_5^{bc} \\ F_6^{bc} \end{Bmatrix} = \begin{bmatrix} k_{33} & k_{34} & k_{35} & k_{36} \\ k_{43} & k_{44} & k_{45} & k_{46} \\ k_{53} & k_{54} & k_{55} & k_{56} \\ k_{63} & k_{64} & k_{65} & k_{66} \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix} = 346.41 \begin{bmatrix} 0 & 0 & 0 & 0 \\ & 1.000 & 0 & -1.000 \\ & & 0 & 0 \\ \text{Sym.} & & & 1.000 \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \end{Bmatrix}$$



Ejemplos

2. Global stiffness equations in matrix form (see Equations 3.5 and 3.6):

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{Bmatrix} = 10^2 \begin{bmatrix} 4.750 & 1.299 & -4.000 & 0 & -0.750 & -1.299 & 0 & 0 \\ & 2.250 & 0 & 0 & -1.299 & -2.250 & 0 & 0 \\ & & 4.750 & 1.299 & 0 & 0 & -0.750 & -1.299 \\ & & & 5.714 & 0 & -3.464 & -1.299 & -2.250 \\ & & & & 4.750 & 1.299 & -4.000 & 0 \\ & & & & & 5.714 & 0 & 0 \\ & & & & & & 4.750 & 1.299 \\ & & \text{Sym.} & & & & & 2.250 \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \Delta_4 \\ \Delta_5 \\ \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix}$$

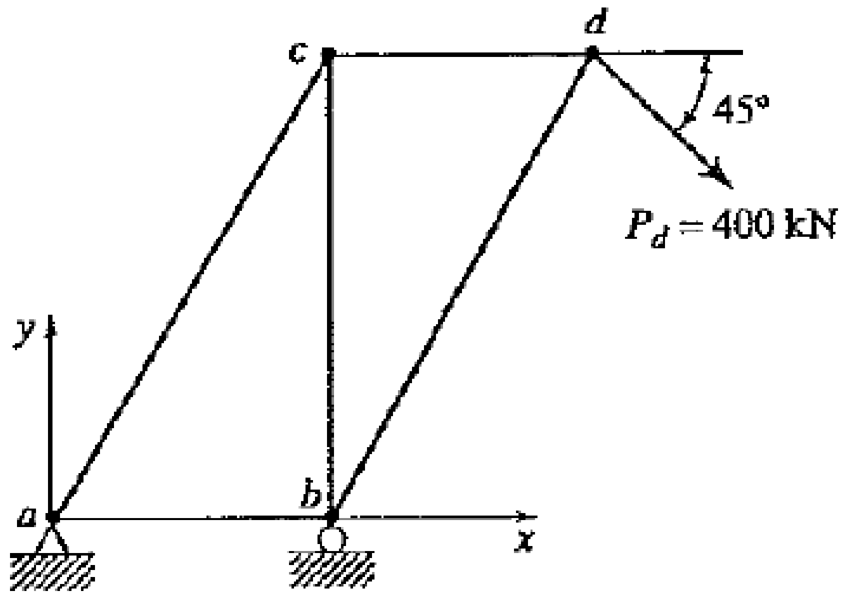
Ejemplos

Problema 3.5

The truss of Example 3.4 is supported and loaded as shown:

1. Calculate the displacements at b, c, and d.
2. Calculate the reactions.
3. Calculate the bar forces.

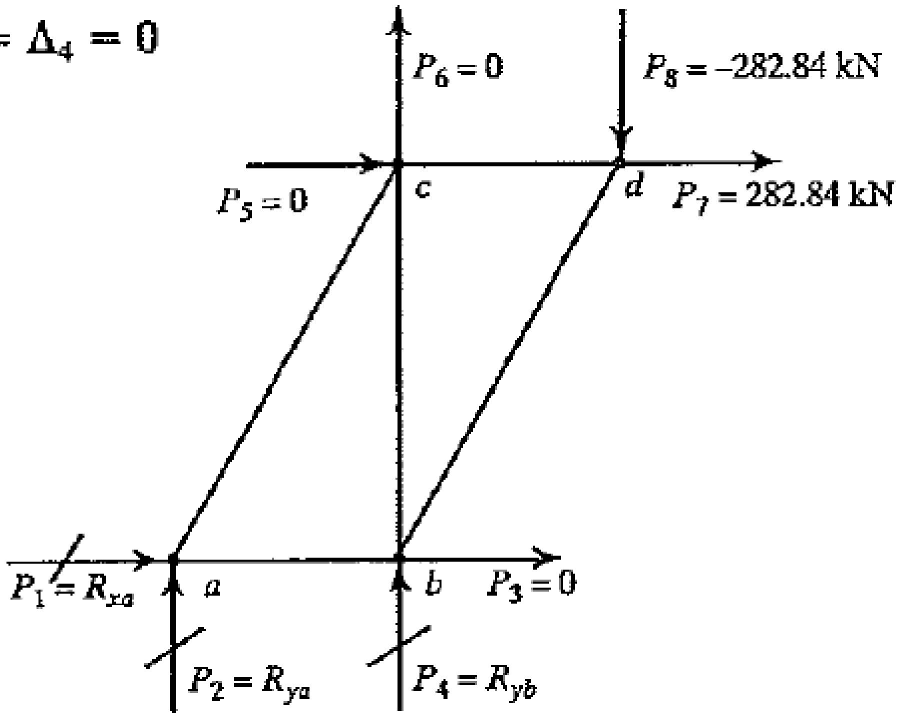
   Use equations of Example 3.4.



Ejemplos

Solución

Boundary conditions: $\Delta_1 = \Delta_2 = \Delta_4 = 0$



Ejemplos

1. Displacements. Remove columns and rows 1, 2, and 4 from the stiffness equations, leaving

$$\begin{Bmatrix} P_3 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 282.84 \\ -282.84 \end{Bmatrix} = 10^2 \begin{bmatrix} 4.750 & 0 & 0 & -0.750 & -1.299 \\ & 4.750 & 1.299 & -4.000 & 0 \\ & & 5.714 & 0 & 0 \\ & \text{Sym.} & & 4.750 & 1.299 \\ & & & & 2.250 \end{bmatrix} \begin{Bmatrix} \Delta_3 \\ \Delta_5 \\ \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix}$$

Solving for $\{\Delta\}$ on a computer yields the following results, which may be checked by substitution in the above equations:

$$\lfloor \Delta \rfloor = \lfloor \Delta_3 \quad \Delta_5 \quad \Delta_6 \quad \Delta_7 \quad \Delta_8 \rfloor = \lfloor -0.407 \quad 9.809 \quad -2.232 \quad 10.926 \quad -7.801 \rfloor \text{ mm}$$

2. Reactions. The remaining stiffness equations (rows 1, 2, and 4) are used as follows:

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_4 \end{Bmatrix} = \begin{Bmatrix} R_{xa} \\ R_{ya} \\ R_{yb} \end{Bmatrix} = 10^2 \begin{array}{c} \begin{array}{ccccc} \Delta_3 & \Delta_5 & \Delta_6 & \Delta_7 & \Delta_8 \end{array} \\ \begin{bmatrix} -4.000 & -0.750 & -1.299 & 0 & 0 \\ 0 & -1.299 & -2.250 & 0 & 0 \\ 1.299 & 0 & -3.464 & -1.299 & -2.250 \end{bmatrix} \end{array} \begin{Bmatrix} -0.407 \\ 9.809 \\ -2.232 \\ 10.926 \\ -7.801 \end{Bmatrix}$$

$$= \begin{Bmatrix} -282.9 \\ -772.0 \\ 1056.2 \end{Bmatrix} \text{ kN}$$

Ejemplos

3. Bar forces. Develop a formula for calculating bar forces from displacements: From equilibrium at the 2 end of a general member 1–2, the bar force F_{12} is

$$F_{12} = F_2 = F_{x2}\cos\phi + F_{y2}\cos(90 - \phi)$$

In matrix form this is

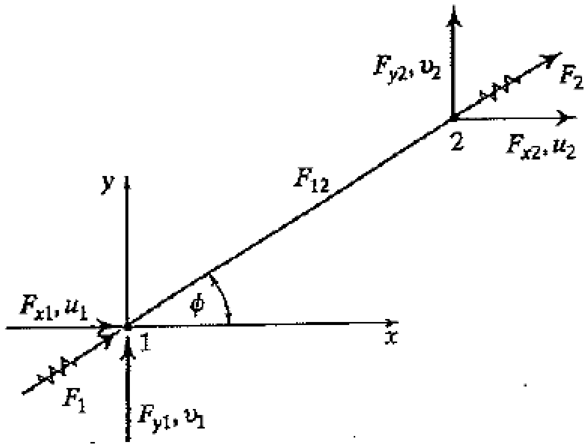
$$F_{12} = \lfloor \cos\phi \quad \sin\phi \rfloor \begin{Bmatrix} F_{x2} \\ F_{y2} \end{Bmatrix} \tag{a}$$

The member stiffness equations are, from Equation 3.11,

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \tag{b}$$

Substituting the last two of Equations b in Equation a gives the desired formula:

$$F_{12} = \lfloor \cos\phi \quad \sin\phi \rfloor \begin{bmatrix} k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix} \tag{c}$$



Ejemplos

Member *ab* $\phi = 0^\circ$

$$F_{ab} = 400.0\begin{bmatrix}1 & 0\end{bmatrix}\begin{array}{cccc} \Delta_1 & \Delta_2 & \Delta_3 & \Delta_4 \\ \end{array}\begin{bmatrix}-1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}0 \\ 0 \\ -0.407 \\ 0\end{Bmatrix} = -162.8 \text{ kN}$$

Member *cd* $\phi = 0^\circ$

$$F_{cd} = 400.0\begin{bmatrix}1 & 0\end{bmatrix}\begin{array}{cccc} \Delta_5 & \Delta_6 & \Delta_7 & \Delta_8 \\ \end{array}\begin{bmatrix}-1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}9.809 \\ -2.232 \\ 10.926 \\ -7.801\end{Bmatrix} = +446.8 \text{ kN}$$

Member *ac* $\phi = 60^\circ$

$$F_{ac} = 300.0\begin{bmatrix}0.500 & 0.866\end{bmatrix}\begin{array}{cccc} \Delta_1 & \Delta_2 & \Delta_5 & \Delta_6 \\ \end{array}\begin{bmatrix}-0.250 & -0.433 & 0.250 & 0.433 \\ -0.433 & -0.750 & 0.433 & 0.750\end{bmatrix}\begin{Bmatrix}0 \\ 0 \\ 9.809 \\ -2.232\end{Bmatrix} = +891.4 \text{ kN}$$

Ejemplos

Member bd $\qquad \phi = 60°$

$$F_{bd} = 300.0\lfloor 0.500 \quad 0.866 \rfloor \begin{array}{cccc} \Delta_3 & \Delta_4 & \Delta_7 & \Delta_8 \\ \left[\begin{array}{cccc} -0.250 & -0.433 & 0.250 & 0.433 \\ -0.433 & -0.750 & 0.433 & 0.750 \end{array}\right] \end{array} \begin{Bmatrix} -0.407 \\ 0 \\ 10.926 \\ -7.801 \end{Bmatrix} = -326.8 \text{ kN}$$

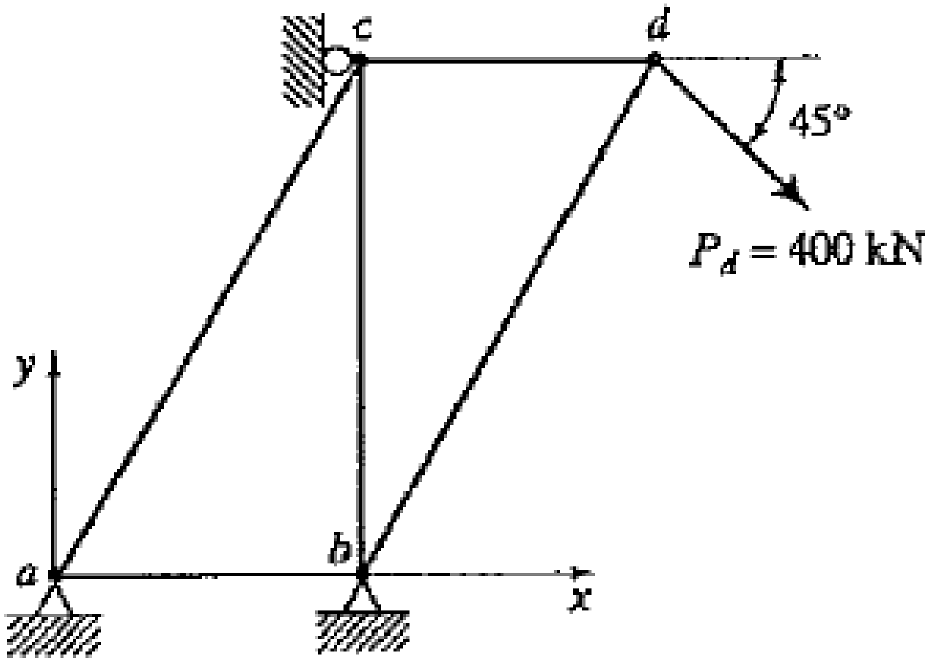
Member bc $\qquad \phi = 90°$

$$F_{bc} = 346.41\lfloor 0 \;\; 1 \rfloor \begin{array}{cccc} \Delta_3 & \Delta_4 & \Delta_5 & \Delta_6 \\ \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{array}\right] \end{array} \begin{Bmatrix} -0.407 \\ 0 \\ 9.809 \\ -2.232 \end{Bmatrix} = -773.2 \text{ kN}$$

Ejemplos

Problema 3.6

The truss shown is the same as in Example 3.5 except for the addition of horizontal constraints at b and c. Calculate the displacements at c and d.

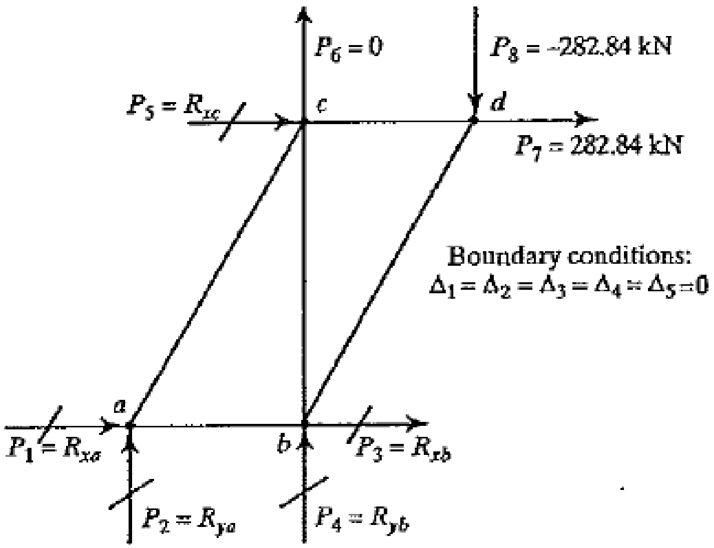


Ejemplos

Solución

Remove columns and rows 1 to 5 from the stiffness equations, leaving

$$\begin{Bmatrix} P_6 \\ P_7 \\ P_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 282.84 \\ -282.84 \end{Bmatrix} = 10^2 \begin{bmatrix} 5.714 & 0 & 0 \\ & 4.750 & 1.299 \\ \text{Sym.} & & 2.250 \end{bmatrix} \begin{Bmatrix} \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix}$$



Solving for $\{\Delta\}$,

$$\begin{Bmatrix} \Delta_6 \\ \Delta_7 \\ \Delta_8 \end{Bmatrix} = 10^{-2} \begin{bmatrix} 0.175 & 0 & 0 \\ & 0.250 & -0.144 \\ \text{Sym.} & & 0.528 \end{bmatrix} \begin{Bmatrix} 0 \\ 282.84 \\ -282.84 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1.114 \\ -1.901 \end{Bmatrix} \text{ mm}$$